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# Al-Fateh University

## Faculty of Engineering

## Electrical and Electronics Engineering Department

EE 303 Numerical Techniques and Programming

Midterm I, November 24th, 2008

- Answer all questions to the best of your knowledge.
- Programmable calculators are not allowed
- No question will be answered during the exam.

Time allowed 90 *minutes*

Q1-

$x^2-3=0 \Rightarrow x=\sqrt{3}$

(a) Use the bisection method to approximate $\sqrt{3}$ to 3 decimal places. Use a=~~1~~ −1 and b=2.(only 4 iterations)

(b) How many iterations are needed for the absolute error in part a of this question to drop down to $10^{-6}$?. What will be the relative error after 19 iterations?

(c) Write Taylor's series expansion of cos(x)

$|b_n - a_n| = (\frac{1}{2})^n (b_0 - a_0)$

**(10 Marks)**

Q2-

(a) Using the secant method find the root of the following: 5 iterations

$f(x) = 3x + sin(x) - e^x$ , Start at $x_0$=1 and $x_1$=0

$x_i$ $x_{i-1}$

(a) Write a C/C++ program for finding the root of a nonlinear equation using secant method

**(10 Marks)**

Q3- Solve the system of equations

5x+ 2y -4z=10
x -2y+ 7z=20
-3x -9y+z=-1

$Ax = b$

(a) By Using LU factorization.

(b) Show that det (A)=det(L)*det(U)

48.6

**(10 Marks)**

$L^{-1}LUX = L^{-1}b$

*Good luck to all of you*

$U^{-1}UX =$

Hannah Montana

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Q1 : $x=\sqrt{3} \Rightarrow x^2-3=0 \Rightarrow f(x)=x^2-3$

(a) $[a,b]=[-1,2]$

$f(a_1)=(-1)^2-3=-2$ , $f(b)=2^2-3=1$ , the root lies betw $a_0$ & $b_0$

$c_1=\frac{-1+2}{2} \Rightarrow \boxed{c_1=0.5}$ , $f(c_1)=-2.75$

$\because f(a_1).f(c_1)>0 \Rightarrow a_2=c_1=0.5$ , $f(a_2)=-2.75$ -ve
$b_2=b_1=2$ , $f(b_2)=1$ +v

$\boxed{c_2=\frac{0.5+2}{2}=1.25}$ , $f(c_2)=(1.25)^2-3 \doteq -1.4375$ -ve

$\therefore a_3=c_2=1.25$ , $f(a_3)=-1.4375$ -v
$b_3=b_2=2$ , $f(b_3)=1$ +v

$c_3=\frac{1.25+2}{2} \Rightarrow \boxed{c_3=1.625}$ , $f(c_3)=(1.625)^2-3$
$f(c_3)=-0.3594$ -ve

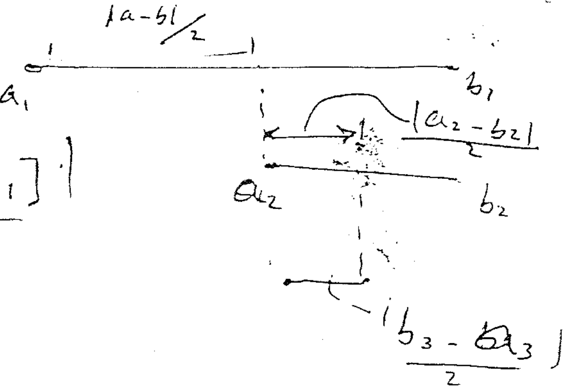
$\therefore a_4=c_3=1.625$
$b_4=b_3=2$ $\Rightarrow c_4=\frac{1.625+2}{2}$

$\boxed{\therefore c_4=1.8125}$

$f(c_4)=(1.8125)^2-3=0.285$

As you see $|f(c_i)|$ decreases but very slowly

(b) Because the interval $[a,b]$ is halved in each iteration so

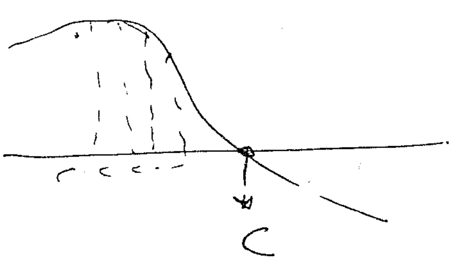



$$\text{Absolute True Error} \leq \frac{|[b_1 - a_1]|}{2^n}$$

$[a_1, b_1] \triangleq$ initial interval

$n \triangleq$ N.O iterations

Absolute True Error of the root $|C - C_n|$




$$10^{-6} \leq \frac{2-(-1)}{2^n}$$

بأخذ log اللوغريتم للأساس 2.

$\therefore$

$$\log_2(10^{-6}) \leq \log_2(3) - \log_2(2^n)$$

$$-6 \cdot \log_2(10) \leq \log_2(3) - n \cdot \log_2(2)$$

$\log_2(2) = 1$

$\log_2(3) = 1.58496$

$\log_2(10) = 3.321928$

$\therefore n \times 1 \leq 1.58496 + 6 \times 3.321928$

$\therefore n \leq 21.516$

$\boxed{\therefore n = 22}$

دائما نقرب للعدد الصحيح الأكبر

if $n \leq 21.001$
$n = 22$

i.e after 22 iterations the absolute true error is less than $10^{-6}$

The Log Function in the Calculator is Logarithm of the 10-base

So

$$\boxed{\log_2(x) = \frac{\log(x)}{\log(2)}}$$

$\log(x) = \log_{10}(x)$

$\log(2) = \log_{10}(2)$

$\log(10) = \log_{10}(10) = 1$

(c) ATE $\leq \frac{3}{2^{19}} = 5.7 \times 10^{-5}$

Q3 $f(x) = 3x + \sin(x) - e^{x}$ , $x_0 = 1$, $x_1 = 0$ , $n = 5$

Rounding off after 5 digits :-

$$\because x_{i+1} = x_i - \frac{f(x_i)\cdot(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

$i = 1$

$x_{i-1} = 1 \Rightarrow f(x_{i-1}) = f(1) = 1.12319$

$x_i = 0 \Rightarrow f(x_i) = f(0) = -1$

$$\therefore x_{i+1} = 0 - (-1)\frac{\cancel{(1.12319)}(1-0)}{(1.12319 - (-1))} \Rightarrow \boxed{x_{i+1} = x_2 = 0.47099}$$

$$\boxed{f(x_{i+1}) = f(0.47099) = 0.26516}$$

$i = 2$ :-

$x_{i-1} = 0$, $f(x_{i-1}) = -1$

$x_i = 0.47099$ , $f(x_i) = 0.26516$

$$x_{i+1} = 0.47099 - \frac{0.26516\,(0 - 0.47099)}{(-1 - 0.26516)}$$

$$\boxed{\therefore x_{i+1} = 0.37228 \,,\; f(x_{i+1}) = f(0.37228) = 0.02953}$$

$i = 3$ :-

$x_{i-1} = 0.47099$, $f(x_{i-1}) = 0.26516$

$x_i = 0.37228$, $f(x_i) = 0.02953$

$$x_{i+1} = 0.37228 - 0.02953\frac{(0.47099 - 0.37228)}{(0.02953 - 0.26516)}$$

$\therefore x_{i+1} =$ [illegible]599 , $f(x_{i+1}) = -0.00129$

لاحظ ان قيمة $|f(x_{i+1})|$ دائما تصغر (تقترب من الصفر) هذا يعني أنها تقترب من الجذر

Convergence

$l_{31} \cdot U_{12} + l_{32} \cdot U_{22} = a_{32}$

$\therefore \cancel{l_{32} = \frac{a_{32} - l_{31} U_{12}}{U_{22}}}$

$l_{32} = \frac{a_{32} - l_{31} \cdot U_{12}}{U_{22}} = \frac{-9 - (-0.6) \times 2}{-2.4} \Rightarrow \boxed{l_{32} = 3.25}$

$l_{31} \cdot U_{13} + l_{32} \cdot U_{23} + U_{33} = 1 \Rightarrow U_{33} = 1 - l_{31} \cdot U_{13} - l_{32} \cdot U_{23}$

$\therefore U_{33} = 1 - (-0.6)(-4) - 3.25(7.8)$

$\boxed{\therefore U_{33} = -26.75}$

$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 0.2 & 1 & 0 \\ -0.6 & 3.25 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 5 & 2 & -4 \\ 0 & -2.4 & 7.8 \\ 0 & 0 & -26.75 \end{bmatrix}$

$\therefore LUX = B$

$L(Z) = B \qquad , \quad Z = UX$

$\therefore \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ -1 \end{bmatrix}$

$\boxed{Z_1 = 10}$ , $l_{31} \cdot Z_1 + Z_2 = 20 \Rightarrow Z_2 = 20 - (+0.2)(10)$

$\boxed{Z_2 = 18}$ , $l_{31} \cdot Z_1 + l_{32} Z_2 + Z_3 = -1 \Rightarrow \cancel{Z_3 = -1 - 0.2(10)} -$
$\cancel{Z_3 = -1 - (-0.6)(10) - (3.25)(20)}$

$-0.6 \times 10 + 3.25 \times 18 + Z_3 = -1$
$\Rightarrow Z_3 = -1 + 6 - 3.25 \times 18$

$\boxed{Z_3 = -53.5}$

$\therefore UX = Z$

$i = 4$

$x_{i-1} = 0.37228, \; f(x_{i-1}) = 0.02953$

$x_i = 0.3599, \; f(x_i) = -0.00129$

$$x_{i+1} = 0.3599 - (-0.00129)\frac{(0.37228 - 0.3599)}{(0.02953 - (-0.00129))}$$

$$\therefore \boxed{x_{i+1} = 0.36042, \; f(x_{i+1}) = 0.00000553}$$

$i = 5$

$x_{i-1} = 0.3599, \; f(x_{i-1}) = -0.00129$

$x_i = 0.36042, \; f(x_i) = 0.00000553$

$$x_{i+1} = 0.36042 - 0.00000553\frac{(0.3599 - 0.36042)}{-0.00129 - 0.00000553}$$

$$x_{i+1} \cong 0.3600$$

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Q 3 :-

$$5x + 2y - 4z = 10$$
$$x - 2y + 7z = 20$$
$$-3x - 9y + z = -1$$

(a) $A = \begin{bmatrix} 5 & 2 & -4 \\ 1 & -2 & 7 \\ -3 & -9 & 1 \end{bmatrix}, \quad L \cdot U = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 5 & 2 & -4 \\ 1 & -2 & 7 \\ -3 & -9 & 1 \end{bmatrix}$$

$u_{11} = a_{11} = 5, \quad u_{12} = a_{12} = 2, \quad u_{13} = a_{13} = -4$

$l_{21} u_{11} = a_{21} \Rightarrow \boxed{l_{21} = \frac{a_{21}}{u_{11}} = \frac{1}{5} = 0.2}$

$l_{21} u_{12} + u_{22} = a_{22} \Rightarrow u_{22} = -2 - (0.2 \times 2) \Rightarrow \boxed{u_{22} = -2.4}$

$l_{21} u_{13} + u_{23} = a_{23} \Rightarrow u_{23} = 7 - (0.2)(-4)$

$\boxed{u_{23} = 7.8}$

$l_{31} u_{11} = a_{31} \Rightarrow \boxed{l_{31} = \frac{-3}{5} = -0.6}$

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$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 5 & 2 & -4 \\ 0 & -2.4 & 7.8 \\ 0 & 0 & -26.75 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 18 \\ -53.5 \end{bmatrix}$$

$$\therefore \; -26.75\, x_3 = -53.5 \Rightarrow \boxed{x_3 = 2}$$

$$-2.4\, x_2 + 7.8\, x_3 = 18 \Rightarrow x_2 = \frac{18 - 7.8 \times 2}{-2.4} \Rightarrow \boxed{x_2 = -1}$$

$$5x_1 + 2x_2 - 4x_3 = 10 \Rightarrow x_1 = \frac{10 - 2(-1) + 4(2)}{5} \Rightarrow \boxed{x_1 = 4}$$

$$\boxed{\begin{aligned} \therefore\; x &= 4 \\ y &= -1 \\ z &= 2 \end{aligned}}$$

substitute $x, y, z$ in the original Eq.

$L.H.S_1 = 5(4) + 2(-1) - 4(2) = 10 = R.H.S_1$

$L.H.S_2 = 4 - 2(-1) + 7(2) = 20 = R.H.S_2$

$L.H.S_3 = -3(4) - 9(-1) + 2 = -1 = R.H.S_3$

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(b) $\because A = L \cdot U \Rightarrow \det(A) = \det(L) \cdot \det(U)$

$\det(L) = 1 \Rightarrow \det(A) = \det(U)$

check by finding $\det(A)$ [illegible] $\det(U)$ for the given

[illegible]